1801. Find the intersections in terms of k, and calculate a definite integral. Assume $k > 0$ to begin with, and then consider the $k < 0$ case.

1802. Use the binomial expansion.

1803. (a) This is like a ladder problem. Draw a force diagram and take moments around the base.
(b) New scenario, new force diagram. Consider the clockwise turning effect of the horizontal frictional force that the van floor must exert on the sheet of plywood.

1804. Find a counterexample. Any non-constant GP and any linear function involving addition of a constant will do the job.

1805. Use the chain rule to differentiate $(\cos x)^{-1}$, or quote a standard result.

1806. This is a coordinate geometry problem involving circles. It boils down to showing that the relevant circles don't intersect.

1807. Make the square root the subject of the equation, and then square both sides.

1808. In algebra, the same operation/function must be applied to both sides, as in op(LHS) = op(RHS). Only in certain cases, such as multiplication by a constant, can this be distributed term by term.

1809. Solve $y^3 - y = 0$.

1810. Consider the fact that the mean is a measure of central tendency, while the variance is a squared measure of spread.

1811. Remember that $\sqrt[4]{x}$ means "the positive fourth root of x".

1812. Factorise the quadratic in $\tan 2x$. There are five roots in the given set.

1813. (a) Differentiate and use the factor theorem.
(b) Use the SPs and the fact that the curve is a positive quartic.

1814. Since chairperson and secretary are different roles, this isn't about ${}^n\mathrm{C}_r$. Simply pick the people for the roles one after another.

1815. Use the factor theorem.

1816. (a) Use Pythagoras.
(b) Unfold to make a flat rhombus $AXCB$.

1817. The second statement is false.

1818. (a) Logs are only defined for positive numbers.
(b) Set the second derivative to zero.
(c) The curve is symmetrical in $x = 2$.

1819. (a) Draw a force diagram (as ever!)
(b) Consider moments around the point at which the lines of action of the reaction forces and weight are concurrent.
(c) Do the same, only numerically.

1820. Sketch the graph $(y - x^2)(y - 4) = 0$, using the factor theorem. Then consider what happens to the sign of the LHS as you cross one of the lines.

1821. Such functions f are polynomial of degree 4.

1822. Sketch the scenario and use its symmetry. Every pair of lines has two angle bisectors, one in the acute angle between them, and one in the obtuse angle. Note that the upper signs in $\pm$ and $\mp$ go together, and so with the lower.

1823. (a) Substitute the exact trig value and rearrange.
(b) Consider the fact that $\frac{\pi}{6}$ is not a particularly small angle.

1824. On a diagram, draw in a radius, and set its length to 1. Then, using exact trig values, find the area of the circumscribed triangle.

1825. (a) The possibility space has $6 \times 12 = 72$ outcomes.
(b) This is simply a calculation of conditional probability, combined with the expectation $\mathbb{E}(X) = np$ of a binomial distribution.

1826. The equation is a hyperbola. The inequality is the interior of a ellipse; its boundary equation is a ellipse. Show algebraically that the two don't intersect.

1827. Consider each graph as one or two reflections of the original graph. Then consider whether the graph must intersect the line of symmetry.

1828. Consider the boundary cases: what shape should a parallelogram/kite be to ensure maximal area?

1829. (a) Firstly find the shaded area using integration. Then equate a definite integral to half of that.
(b) Use the iteration $k_{n+1} = \frac{1}{3}(k_n^3 + 1)$, starting with $k_0 = 0.5$.

1830. The possibility space is 64 equally likely outcomes. List the successful ones.

1831. (a) Find the y coordinate of each vertex.
(b) It is not guaranteed. Explain why.

1832. According to the reverse chain rule, multiply by the reciprocal of the derivative of the linear inside function. And don't forget the...

1833. (a) Differentiate to show that the gradient is $2p-1$ at $(p, p^2 - p)$. Then use the standard formula $y - y_1 = m(x - x_1)$.
(b) Sub $(6, 14)$ into your generic tangent line.

1834. Begin with $(2z + 1)^2$, and work from there.

1835. Use log rules to consider each of the curves as a translation of $y = \ln x$.

1836. Differentiate implicitly using the chain rule.

1837. (a) Use the sum of an AP. Or, use the fact that the sum of the first n integers is $\frac{1}{2}n(n + 1)$.
(b) Subtract the sum of the multiples of four from the overall sum.

1838. Restrict the possibility space: you can evaluate both probabilities.

1839. Use the fact that $\log_{a^k} b^k \equiv \log_a b$.

1840. (a) The square roots cancel.
(b) The expression is a difference of two squares.

1841. Consider the isosceles triangle which is formed of two radii and a side. Split this in two, then use trigonometry.

1842. Remember that a sample in the acceptance region signifies *insufficient* evidence (as opposed to none at all) to reject the null hypothesis.

1843. Assume, for a contradiction, that the other factor is ax^2+bx+c. Then compare coefficients, starting with x^4.

1844. (a) Differentiate as normal.
(b) Substitute for x and $\frac{dx}{dt}$ in the DE. Remember that a solution (to a DE) requires that the two sides be *identical*.

1845. Expand the conditional probabilities using the standard formula. You can work on the inequality itself, rather than a boundary equation, because the probabilities are positive.

1846. Use the sine area formula, and the standard $\frac{1}{2}bh$.

1847. (a) The Reuleaux triangle contains the equilateral triangle.
(b) Consider the radii of the arcs.
(c) Calculate the area of three segments in the usual fashion, i.e. as sector minus triangle.

1848. Solve each inequality individually, then consider the intersection of the sets.

1849. The implication goes backwards. Find a specific counterexample to the forwards implication.

1850. Rewrite the logarithmic equation in index form, and compare.

1851. Show that $k < 0$. Then eliminate the case in which the trapezium is also a parallelogram.

1852. Substitute the second equation into the RHS of the result you are trying to reach.

1853. Find the relevant derivative. Then substitute for x and $\frac{dx}{dt}$ in the LHS and simplify.

1854. This is in a sense obvious. Proving an "obvious" result just means making it even more obvious! Consider the cases in which the central square is shaded or not.

1855. (a) Sketch a normal distribution, and think about its curvature (second derivative).
(b) Use the fact that the points of inflection on a normal distribution are at $|X - \mu| = \sigma$.

1856. Consider the multiplicity of the roots/factors and the sign of the leading coefficient.

1857. Substitute for y, and solve a quadratic in $\sqrt{x}$ using the formula. Consider the validity of the roots.

1858. Translations do not affect areas; stretches do.

1859. Use $\cos(x \pm y) \equiv \cos x \cos y \mp \sin x \sin y$. Note the $\mp$ sign on the RHS.

1860. Find the side lengths, and then use the cosine rule, followed by the sine area formula. Or you could use Heron's formula, if you know it.

1861. Solve the inequality algebraically before using the distribution of X to calculate probabilities.

1862. In each case, write the base as a power of 4, then use index laws.

1863. (a) Substitute the definition of $\star$.
(b) You should get a quadratic.

1864. (a) Apply the iteration twice to x_0. To simplify, multiply top and bottom of the main fraction by the denominator of the inlaid fraction.
(b) Using the same technique, find x_3. You should see a pattern.

1865. One is true and the other is false.

1866. Consider a possibility space of ${}^8\text{C}_3$ equally likely outcomes.

——— Alternative Method ———

Place the first vertex arbitrarily, without loss of generality. Then consider the ${}^7\text{C}_2$ equally likely ways in which the remaining two vertices may be chosen.

1867. Set up the standard equation for fixed points, and use the discriminant.

1868. Assume, for a contradiction, that a frequency ratio $k : 1$, where $k > 1$, can be expressed as a whole number of octaves and also as a whole number of fifths. This means that $k = 2^p$ for $p \in \mathbb{N}$ and $k = \frac{3}{2}^q$, where $q \in \mathbb{N}$. Combine these and use prime factors to find the contradiction.

1869. Find intersections, and then set up a single definite integral for the (signed) area.

1870. You can either use a compound-angle formula, or else (and rather slicker) consider the symmetry of the sine and cosine functions as defined on the unit circle.

1871. Set up $\left(p + q\sqrt{2}\right)^2 = 33 + 8\sqrt{2}$. Multiply this out and equate coefficients.

1872. Use the product rule.

1873. Sketch the curve, and consider its reflection in $y = x$.

1874. Consider the case in which the solution set S is the empty set.

1875. (a) Use log laws.
(b) The mean and median of an AP are the same: the mean of the first and last terms.

1876. Draw clear force diagrams. You might consider the extreme cases of no friction and total friction.

1877. Consider whether the denominator can be zero.

1878. This is the factor theorem. Substitute a value to show that $(3x+5)$ is not a factor of the numerator.

1879. Solve for x_1 in $\mathbb{P}(X < x_1) = 0.1$, and set up a similar equation for x_2.

1880. The sum is simply a polynomial in x. Write it out longhand, and integrate as usual.

1881. Set the outermost square to have side length 1, and calculate the total shaded area as $\frac{1}{2} + \ldots$. Then use $S_\infty = \frac{a}{1-r}$.

1882. Determine which variable you don't want, and eliminate it using substitution.

1883. At a point of inflection, the second derivative must change sign. It is not sufficient that the second derivative is zero.

1884. You might find a Venn diagram useful here. The backslash is the "set minus" notation, so that $A \backslash B$ is the same as $A \cap B'$.

1885. Consider the equivalent question with one linear equation in two unknowns.

1886. (a) Draw a force diagram.
(b) Consider the acceleration of the system.

1887. The sector formed when a cone is unwrapped has an arc length defined by the circumference of the base of the cone. A "right-circular" cone is just a standard cone.

1888. Consider the range of $1 \pm \text{g}(x)$.

1889. Use the standard formula for the partial sum of an AP, and solve a quadratic in n.

1890. In each case, consider the degree of the polynomial equation satisfied by the intersections.

1891. Standard deviation scales inversely with the square root of sample size.

1892. (a) Draw a sketch and use trigonometry.
(b) Divide the equations from part (a).
(c) Divide both sides by $\sin\theta$.

1893. Integrate the second derivative, and use the value of $\text{f}'(2)$ to find the $+c$. Then solve $\text{f}'(x) = 0$.

1894. Either use $\sin^2 x + \cos^2 x \equiv 1$, or $\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$.

1895. The minimum speed will be attained at 45° above the horizontal.

1896. Assume that you do have such constants, and work to find a contradiction. Start by multiplying up by the denominators.

1897. Calculate the area of the segment using the usual method of sector minus triangle. Each of these should be in terms of r and θ.

1898. Write 2 over base e, then use an index law and the chain rule.

1899. Find a weighted average of the scale factors of the unchanged part and the changed part.

1900. Multiply up, then use log rules to manipulate the equation into the form $\ln a = \ln b$. At that point, you can exponentiate both sides. Remember to check the validity of any roots you find.

——— End of 19th Hundred ———

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